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APPROXIMATION IN A DIFFERENTIAL GAME

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We examine the solution of a game problem of encounter [1] and we investigate its stability with respect to errors in the measurement of the game's current position. We describe a modification of the extremal strategy, guaranteeing a stable encounter with the target. We present examples illustrating the proposed control method.

Let us consider the controllable system

$$x^{\cdot} = f(t, x, u, v) \tag{1}$$

Here x is the phase vector; u, v are the controls of the first and second players, subject to the constraints

$$u \in P, \qquad v \in Q \tag{2}$$

where P and Q are compacta. We accept that the function f is continuous in all arguments and satisfies the local Lipschitz condition in x and the inequality

$$\| f(t, x, u, v) \| \leqslant \varphi (1 + \| x \|)$$

where $\| x \|$ is the Euclidean norm of vector x; φ is a constant.

We shall solve the first player's game problem of encounter [1] with a set M under the condition $\{t, x[t]\} \in N$. Here M and N are given closed sets in the space of $\{t, x\}$. We examine the problem in the framework of mixed strategies $U \div \mu(du; t, x)$ and $V \div \nu(dv; t, x)$, treating them and the motions $x[t]$ generated by them as in [1]. We note that in [1] the player's mixed strategies U and V are identified with nonsingle-valued transformations of the $\{t, x\}$-space onto sets of probability measure $\{\mu(du)\}$ and $\{\nu(dv)\}$, normed on compacta P and Q, respectively, while here the strategies are defined as single-valued functions $U \div \mu(du; t, x)$ and $V \div \nu(dv, t, x)$. However, the difference indicated is insignificant, and all the results of [1] remain valid for the single-valued strategies to be considered here.

From [1] it follows that in order to solve the given problem of the encounter of all motions $x[t] = x[t, t_0, x_0, U^0]$ with M inside N not later than at some instant $\vartheta > t_0$, it is sufficient to construct a closed set $W^{(\vartheta)} \subset N$, u-stable relative to M, containing the initial position $\{t_0, x_0\}$ and terminating on M at the instant ϑ. In what follows, this set

$$W^{(\vartheta)}: [W(t, \vartheta), \ t_0 \leqslant t \leqslant \vartheta] \tag{3}$$

is called a stable bridge in the $\{t, x\}$-space. In particular, according to [1], the role of the stable bridge $W^{(\vartheta)}$ can be played by the set W of position absorption of target M at the instant ϑ. The strategy $U_e \div \mu_e(du; t, x)$, extremal to the stable bridge

$W^{(\vartheta)}$ also is that strategy U^0, which solves the problem.

These formal assertions are meaningfully revealed in the stochastic approximation scheme forming the random motions $x_\Delta[t, t_0, x_0, U^0]$ which for a sufficiently small step $\delta = \sup_i (\tau_{i+1} - \tau_i)$ $(i = 0, 1, \ldots)$ encounter, with a probability p arbitrarily close to one, the ε-neighborhood of M inside the ε-neighborhood of N at the instant ϑ, where $\varepsilon > 0$ can be chosen arbitrarily small [2, 3]. Here $[\tau_i, \tau_{i+1})$ $(i = 0, 1, \ldots)$ are half-open intervals on which the random controls $u[t]$ are constant. However, if the quantity $x_\Delta[\tau_i]$ determining the control $u[t] = u[\tau_i]$ $(\tau_i \leqslant t < \tau_{i+1})$ were to enter the controller with an error, then for the stable operation of the scheme mentioned we would need, in certain nonregular cases, a further constraint from below on the step δ of the approximation scheme. In this case the constraints of the measurement error in $x_\Delta[\tau_i]$ may prove to be excessive. The simplest example of such a nonregular situation is furnished by the problem of the encounter of a point $\xi[t]$, moving along the straight line $-\infty < \xi < \infty$ and described by the equation

$$\xi^\cdot = u - v, \quad |u| \leqslant 2, \quad |v| \leqslant 1 \tag{4}$$

with any of two points $\xi^{(1)} = -1$, $\xi^{(2)} = 1$. The problem is solved by the control $u(\xi) = 2$ when $\xi \geqslant 0$ and $u(\xi) = -2$ when $\xi < 0$. However, when we pass to the descrete scheme, in the case when the quantity $\xi[\tau_i]$ is introduced into the control $u[t] = u(\xi[\tau_i])$ $(\tau_i \leqslant t < \tau_{i+1})$ with an error $\Delta\xi$ which can exceed the amount $\alpha = {}^1/_2\,\delta = {}^1/_2 \sup_i (\tau_{i+1} - \tau_i)$, the point $\xi[t]$ can get trapped in a neighborhood of the point $\xi = 0$.

The purpose of the present note is to indicate a small modification of the approximation scheme mentioned [2, 3] for forming the random motions $x_\Delta[t]$, which permits us to by-pass the circumstance stated. The essence of this modification is that instead of the extremal control $\mu_e(du; \tau_i, x_\Delta[\tau_i])$ which aims the motion $x_\Delta[t]$ at the position $\{\tau_i, x_\Delta[\tau_i]\}$ towards the nearest point $w^0[\tau_i]$ of $W(\tau_i, \vartheta)$, we shall introduce a control $\mu_w(du; \tau_i, x_\Delta[\tau_i], w_\Delta[\tau_i])$ which aims the motion $x_\Delta[t]$ towards some also sufficiently close point $w_\Delta[\tau_i]$ of $W(\tau_i, \vartheta)$, but now not necessarily the point $w^0[\tau_i]$ of $W(\tau_i, \vartheta)$, closest to $x_\Delta[\tau_i]$. Let us describe this control.

We examine two motions. The driven motion $x_\Delta[t]$ in the given actual controlled system, described by Eq. (1), and the driving motion $w_\Delta[t]$ produced by a precision model of suitable degree and described by the equation

$$w_\Delta^\cdot[t] = \int_P \int_Q f(t, w_\Delta[t], u, v)\, \mu_t(du)\, \nu_t(dv) \tag{5}$$

In accordance with the problem statement the control u in Eq. (1) is prescribed by the first player and the control v, by the second player. In Eq. (5) both "controls" μ and ν are prescribed by the first player. Thus, we describe the construction of the control $u[t]$ for the motion $x_\Delta[t]$ and of the controls $\mu_t(du)$ and $\nu_t(dv)$ for the motion $w[t]$.

Suppose that for a given initial position $\{t_0, x_0\}$ we have succeeded in finding the stable bridge $W^{(\vartheta)}$, containing this position, lying in N and terminating on M at the instant $t = \vartheta$. Having chosen the partitioning $\Delta : [\tau_i, i = 0, 1, \ldots; \tau_0 = t_0]$ of the semiaxis $[t_0, \infty)$, we form the two motions $x_\Delta[t]$ and $w_\Delta[t]$ in the following manner. At the initial instant $t = t_0 = \tau_0$ we set $w_\Delta[t_0] = x_\Delta[t_0] = x_0$ and we choose the control $u[t] = u[\tau_0] \in P$ arbitrarily on the half-open interval $[\tau_0, \tau_1)$. We choose the measure $\nu(dv; \tau_0)$ arbitrarily, normed on Q, and we determine the mixed program control $\mu_t(du;\,\tau_0)$ $(\tau_0 \leqslant t < \tau_1)$ such that for the motion $w_\Delta[t]$ satisfying the eqaution

$$w^{\boldsymbol{\cdot}}_\Delta[t] = \iint f(t, w_\Delta[t], u, v)\,\mu_t(du;\,\tau_0)\,\nu(dv, \tau_0)$$
$$(w_\Delta[t_0] = x_0,\ \ \tau_0 \leqslant t < \tau_1) \tag{6}$$

one of the conditions

$$\{\tau_1,\ \ w_\Delta[\tau_1]\} \in W^{(\vartheta)} \tag{7}$$

or

$$\cup\ [\{t, w_\Delta[t]\},\ \ \tau_0 \leqslant t \leqslant \tau_1] \cap M \neq \phi$$

is fulfilled. The possibility of choosing such a control $\mu_t(du)$ follows from the condition for the stability of bridge $W^{(\vartheta)}$. We note that in [1] the stability condition implies the existence of a motion $w[t]$ satisfying the contingent equation

$$w^{\boldsymbol{\cdot}}[t] \in \mathrm{co}\left[\int_Q f(t, w[t], u, v)\,\nu(dv;\,\tau_0);\, u \in P\right] \tag{8}$$

and one of the conditions (7). However, it can be shown that by an appropriate definition of the class of mixed program controls $\mu_t(du)$ (see [4]) every solution of the contingent equation (8) is a solution of an equation of form (6).

Now suppose that at the instant $t = \tau_i$ $(i = 1, 2, \ldots)$ we have realized the points $x_\Delta[\tau_i]$, $w_\Delta[\tau_i]$, and that the driving motion $w_\Delta[t]$ for $t_0 \leqslant t \leqslant \tau_i$ has not hit onto set M. Suppose that information on the realized value $x_\Delta[\tau_i]$ has been fed into the first player's controller in the form of a signal $x_\Delta{}^*[\tau_i]$ related with the value $x_\Delta[\tau_i]$ by the inequality

$$\| x_\Delta[\tau_i] - x_\Delta{}^*[\tau_i] \| \leqslant \zeta \tag{9}$$

We construct the vector $s = x_\Delta{}^*[\tau_i] - w_\Delta[\tau_i]$ and we consider a small game in which the quantity $s'f(\tau_i, x_\Delta{}^*[\tau_i], u, v)$ (the prime denotes transposition) serves as the cost, i.e. we consider the problem of determining the measures $\mu^0(du;\,\tau_i)$ and $\nu^0(dv;\,\tau_i)$ which satisfy the condition (see [1])

$$\iint_{PQ} s'f(\tau_i, x_\Delta{}^*[\tau_i], u, v)\,\mu^0(du;\,\tau_i)\,\nu(dv) \leqslant$$
$$\iint_{PQ} s'f(\tau_i, x_\Delta{}^*[\tau_i], u, v)\,\mu^0(du;\,\tau_i)\,\nu^0(dv;\,\tau_i) \leqslant$$
$$\iint_{PQ} s'f(\tau_i, x_\Delta{}^*[\tau_i], u, v)\,\mu(du)\,\nu^0(dv;\,\tau_i) \tag{10}$$

We choose the control $\mu_t(du;\,\tau_i)$ $(\tau_i \leqslant t < \tau_{i+1})$ such that one of the two conditions

$$\{\tau_{i+1},\ \ w_\Delta[\tau_{i+1}]\} \in W^{(\vartheta)} \tag{11}$$

or

$$\cup\ [\{t, w_\Delta[t]\};\ \tau_i \leqslant t \leqslant \tau_{i+1}] \cap M \neq \phi$$

is fulfilled for a motion $w_\Delta[t]$ satisfying the equation

$$w_\Delta{}^{\boldsymbol{\cdot}}[t] = \iint_{PQ} f(t, w_\Delta[t], u, v)\,\mu_t(du;\,\tau_i)\,\nu^0(dv;\,\tau_i)$$
$$(\tau_i \leqslant t < \tau_{i+1})$$

As above, the possibility of choosing such a control $\mu_t(du;\,\tau_i)$ follows from the condition for the stability of the bridge $W^{(\vartheta)}$. We choose the control

$u[t] = u[\tau_i]$ $(\tau_i \leqslant t < \tau_{i+1})$ for the motion $x_\Delta[t]$ from the result of a random experiment with probability distribution $\mu^0(du;\ \tau_i)$.

The described approximation scheme for choosing the first player's random controls $u[t] = u[\tau_i]$ $(\tau_i \leqslant t < \tau_{i+1},\ i = 0, 1, \ldots)$ is realized in system (1) in pair with an arbitrary deterministic or random control of the second player. Here it is assumed that the realizations of controls $u[t]$ and $v[t]$ are stochastically independent. The presence of measurement error in the phase vector $x_\Delta[t]$ can serve as the physical premise for such an assumption. Indeed, if the error estimate, namely, the number ζ for the second player, is greater than the quantity $\lambda\delta$, where λ is a Lipschitz constant and δ is the step in the approximation scheme being considered, then the second player is not able to reestablish the piecewise-constant control chosen by the first player. We note that the controls $u[t] = u[\tau_i]$ $(\tau_i \leqslant t < \tau_{i+1})$ are also chosen independently of the control $v[t]$. If the distributions of the error Δx are considered, these arguments can be framed in rigorous concepts. The following assertion is valid.

Theorem. Whatever be $\varepsilon > 0$ and $p < 1$, we can find arbitrarily small numbers $\zeta_{\varepsilon,p} > 0$ and $\delta_{\varepsilon,p} > 0$ such that when the inequalities $\zeta \leqslant \zeta_{\varepsilon,p}$ and $\delta \leqslant \delta_{\varepsilon,p}$ are fulfilled for the motions $x_\Delta[t]$, contact with the ε-neighborhood of M inside the ε-neighborhood of N at the instant ϑ is guaranteed with a probability not less than p.

The approximation procedure described above corresponds to the theoretical constructions which were presented in [1, 2]. The theorem id proved by the plan proposed in these papers, with this difference that here we estimate not the distance from $x_\Delta[t]$ to the set $W^{(\vartheta)}$ or M but the distance from $x_\Delta[t]$ to the point $w_\Delta[t]$ which, moving along the bridge $W^{(\vartheta)}$, inevitably hits onto M for $t \leqslant \vartheta$.

If at every position $\{t, x\}$ and for every choice of vector s the small game (10) has a saddle point in the pure strategies u^0 and v^0, i.e. the equality

$$\min_{u\in P}\max_{v\in Q} s'f(t,x,u,v) = \max_{v\in Q}\min_{u\in P} s'f(t,x,u,v) \qquad (12)$$

is fulfilled for all values of $t \leqslant \vartheta$, x and s, then the motion $x_\Delta[t]$ is obtained as deterministic, and the Theorem's assertion on the contact of $x_\Delta[t]$ with the ε-neighborhood of M inside the ε-neighborhood of N for $t \leqslant \vartheta$ reduces to the fact that this contact will take place with certainty. In this case the assumption of mutual independence of the player's choice of controls can be dropped and such methods of forming v are allowed the opprонent, which use information of the first player's realized control.

If, however, we are required to find a deterministic solution of a position encounter problem in the case when equality (12) is violated, then, in accordance with the results in [5, 6], we can suggest the following modification of the approximation procedure described above. Suppose that for a given initial position $\{t_0, x_0\}$ we have succeeded in finding a minimax u-stable bridge (3) (see [6]). At the initial instant $t = t_0$ we set $x_\Delta[t_0] = w_\Delta[t_0] = x_0$. We assume that the point $\{t, w_\Delta[t]\}$ has not hit onto set M for $t \in [t_0, \tau_i]$, then, for constructing the motions $x_\Delta[t]$ and $w_\Delta[t]$ for $\tau_i \leqslant t < \tau_{i+1}$, we proceed as follows. We construct a vector $s = x_\Delta^*[\tau_i] - w_\Delta[\tau_i]$, where, as above, $x_\Delta^*[\tau_i]$ is the signal of the vector $x_\Delta[\tau_i]$ fed to the first player. We consider a small game in the class of pure strategies $u \in P$ and counterstrategies $v(u) \in Q$, i.e. we determine the vector $u^0[\tau_i] \in P$ and the vector-valued function $v^0(u;\tau_i)$ satisfying the condition

$$s'f(\tau_i, x_\Delta^*[\tau_i], u^0[\tau_i], v) \leqslant s'f(\tau_i, x_\Delta^*[\tau_i], u^0[\tau_i], v^0(u^0[\tau_i]; \tau_i)) \leqslant$$
$$s'f(\tau_i, x_\Delta^*[\tau_i], u, v^0(u, \tau_i))$$

We define the driving motion $w_\Delta[t]$ for $\tau_i \leqslant t < \tau_{i+1}$ as the solution of the contingent equations

$$w_\Delta^\cdot[t] \in \mathrm{co}\,[f(t, w_\Delta[t]\, u, v^0(u, \tau_i)), u \in P]$$

satisfying one of conditions (11). The existence of such a motion follows from the condition of minimax u-stability of bridge $W^{(\vartheta)}$. The driven motion $x_\Delta[t]$ for $\tau_i \leqslant t < \tau_{i+1}$ is generated by the constant control $u[t] = u^0[\tau_i]$ and by a certain realization of the second player's controlling action. Here the formation of control $v[t]$ which uses the information on the control $u[t]$ realized, is also allowed.

Now an assertion analogous to the Theorem is valid once again, i.e. all motions $x_\Delta[t]$ constructed by the method indicated, encounter the ε-neighborhood of M inside the ε neighborhood of N at the instant ϑ for sufficiently small ζ and δ.

We note that when $i = 0$ the vector s coincides with the measurement error of vector $x_\Delta[t_0]$, i.e. $s = x_\Delta^*[t_0] - x_\Delta[t_0]$, therefore, just as in the preceding case, the control $u^0[\tau_0] \in P$ can be chosen arbitrarily. In isolated cases the formation of control u by the scheme described can lead to very easily realizable procedures for the control.

For example, let us consider the problem of the evasion (see [7], pp. 328-342) of one motion $z(t)$ from another motion $y[t]$ in the case of linear objects of the same type

$$y^\cdot = Ay + u, \qquad u \in P$$
$$z^\cdot = Az + v, \qquad v \in Q$$

where the convex sets P and Q are similar and P is larger than Q. The contact of the motions is defined as the fulfillment of the condition $(y[\tau] - z[\tau]) \in S$, where S is some given closed set. The given problem of evasion up to an instant ϑ can be treated with an interchange of the symbols u and v as the encounter problem considered in this paper if as M we take the hyperplane $t = \vartheta$ and as N, the complete halfspace $\{t, x\} = \{t, y - z\}$ for $t \leqslant \vartheta$ less a certain suitable open region containing the set $\{t < \vartheta, (y - z) \in S\}$. It then turns out that the control $v = v[\tau_i] \in Q$ for the real motion $x_\Delta[t] = y[t] - z_\Delta[t]$ is determined by the condition

$$(x_\Delta^*[\tau_i] - w_\Delta[\tau_i])' v[\tau_i] = \max_{v\in Q} (x_\Delta^*[\tau_i] - w_\Delta[\tau_i])' v \tag{13}$$

while the controls $u_w[\tau_i]$ and $v_w[\tau_i]$ for the model motion $w_\Delta[t]$, which satisfies the equation

$$w_\Delta^\cdot[t] = Aw_\Delta[t] + u_w - v_w \tag{14}$$

are determined by the conditions

$$(x_\Delta^*[\tau_i] - w_\Delta[\tau_i])' u_w[\tau_i] = \max_{u\in P} (x_\Delta^*[\tau_i] - w_\Delta[\tau_i])' u$$
$$v_w[\tau_i] = \beta u_w[\tau_i] \tag{15}$$

where β is the ratio of the dimensions of set Q to the dimensions of set P.

These conditions have the following simple meaning. Condition (13) aims the real point $x_\Delta[\tau_i]$ at the model point $w_\Delta[\tau_i]$, while conditions (15) ensure only such realizations $\sigma[t]$ of the control $\sigma = u_w - v_w$ which are contained in the set $(\beta - 1)\,Q$. But, according to [7], under such controls the point $w_\Delta[t]$ from (14) cannot encounter the target S earlier than at the instant ϑ which is the optimal pursuit time for the given initial position.

As another example we consider the problem of evading the state $x_1 = x_2 = 0$ for the system described by the following equations [8]:

$$\begin{gathered} x_1^{\cdot} = x_3, \quad x_3^{\cdot} = u_1 \cos v_3 - u_2 \sin v_3 - v_1 \cos u_3 + v_2 \sin u_3 \\ x_2^{\cdot} = x_4, \quad x_4^{\cdot} = u_1 \sin v_3 + u_2 \cos v_3 - v_1 \sin u_3 - v_2 \cos u_3 \\ u_1^2 + u_2^2 \leqslant r_1^2, \quad v_1^2 + v_2^2 \leqslant r_2^2, \quad |u_3| \leqslant \alpha, \quad |v_3| \leqslant \beta \\ \alpha < \pi/2, \quad \beta < \pi/2, \quad r_1 \cos\beta > r_2 \cos\alpha \end{gathered} \tag{16}$$

Relying on the material from [8] we can convince ourselves that the best method of evasion for the given problem in the control scheme described in the present paper is determined by the following conditions:

for the real motion $x_\Delta[t]$

$$v_1[\tau_i] = r_2 s_1[\tau_i] / \| s[\tau_i] \|, \quad v_2[\tau_i] = r_2 s_2[\tau_i] / \| s[\tau_i] \| \tag{17}$$

$v_3[\tau_i] = \pm\beta$ with probability $p\{v_3 = -\beta\} = p\{v_3 = \beta\} = 1/2$

for the model motion $w_\Delta[t]$

$$u_1^{(w)}[\tau_i] = r_1 \cos\beta\, s_1[\tau_i] / \| s[\tau_i] \|, \quad u_2^{(w)}[\tau_i] = r_1 \cos\beta\, s_2[\tau_i] / \| s[\tau_i] \|$$

$$u_3^{(w)} = 0$$

$$v_1^{(w)}[\tau_i] = u_1^{(w)}[\tau_i]\, r_2 \cos\alpha / r_1 \cos\beta \tag{18}$$

$$v_2^{(w)}[\tau_i] = u_2^{(w)}[\tau_i]\, r_2 \cos\alpha / r_1 \cos\beta, \quad v_3^{(w)} = 0$$

where the vector $s = \{s_1, s_2\} = \{x^*_{\Delta,3} - w_{\Delta,3},\ x^*_{\Delta,4} - w_{\Delta,4}\}$. Such is the realization of the approximation scheme for the given problem when we are required to find its solution in the class of stochastic control methods. However, if we are required to find a deterministic solution, relations (17), (18) should be replaced as follows:

$$v_1[\tau_i] = r_2 s_1[\tau_i] / \| s[\tau_i] \|, \quad v_2[\tau_i] = r_2 s_2[\tau_i] / \| s[\tau_i] \|$$

$$v_3[\tau_i] = 0$$

$$u_1^{(w)}[\tau_i] = r_1 s_1[\tau_i] / \| s[\tau_i] \|, \quad u_2^{(w)}[\tau_i] = r_1 s_2[\tau_i] / \| s[\tau_i] \|$$

$$u_3^{(w)}[\tau_i] = 0$$

$$v_1^{(w)}[\tau_i] = u_1^{(w)}[\tau_i]\, r_2 \cos\alpha / r_1$$

$$v_2^{(w)}[\tau_i] = u_2^{(w)}[\tau_i]\, r_2 \cos\alpha / r_1, \quad v_3^{(w)} = 0$$

Here it is assumed that $r_1 > r_2 \cos\alpha$.

Other examples of the effective use of the approximation procedure described are furnished by the cases when the stable bridges $W^{(\vartheta)}$ can be constructed sufficiently simply on the basis of [9]. As an illustration of this situation we can consider the solution of the problem of damping a mathematical pendulum on which acts the force

$$F = u_1 + (u_2 - v)^2 - 1, \quad |u_1| \leqslant 1, \quad |u_2| \leqslant 1, \quad |v| \leqslant 1$$

The realization of the approximation in the given problem, described by the equation

$$x_1^{\cdot} = x_2, \; x_2^{\cdot} = -x_1 + u_1 + (u_2 - v)^2 - 1$$

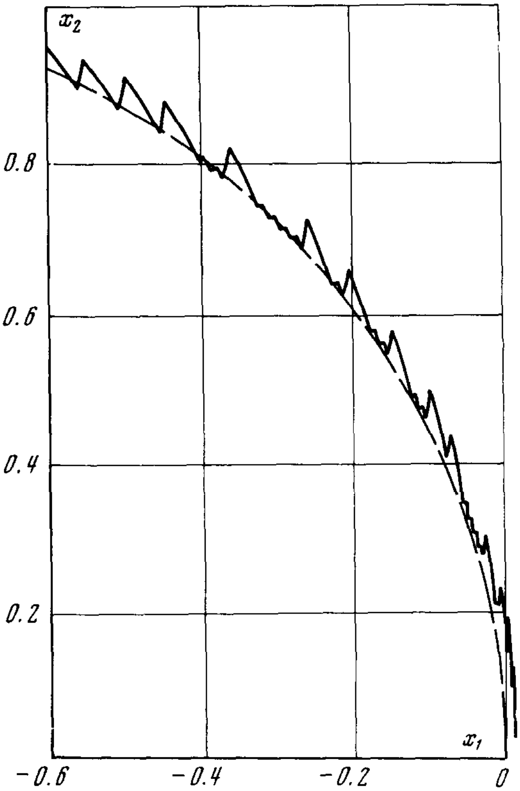

Fig. 1

reduces to the following relations:

for the real motion

$$u_1[\tau_i] = -1, \quad u_2[\tau_i] = 0 \quad \text{if } s_2[\tau_i] \geqslant 0$$

$$u_1[\tau_i] = 1$$

$$p\{u_2 = 1\} = p\{u_2 = -1\} = 1/2 \quad \text{if } s_2[\tau_i] < 0$$

where $s_2[\tau_i] = x^*_{\Delta,2}[\tau_i] - w_{\Delta,2}[\tau_i]$;

for the model motion $w_\Delta[t]$, described by the equation

$$w_1^\cdot = w_2, \quad w_2^\cdot = -w_1 + \sigma, \ |\sigma| \leqslant 1$$

the control $\sigma[t]$ is formed in correspondence with the known solution of the problem of optimal damping of a mathematical pendulum (for example, see [10], pp. 34-42). The point $w_\Delta[t]$ actually moves along a u-stable path $w(t)$ furnished by this solution.

Figure 1 shows one of the sections of the approximate motion $x_\Delta[t]$, the step $\delta = \tau_{i+1} - \tau_i$ is equal 0.01, the control $v[t]$ is chosen in the following manner: $v = 0$ for $s_2 \geqslant 0$, $v = \pm 1$ for $s_2 < 0$, the values +1 and − 1 are chosen with equal probability; the dashed line represents the motion $w_\Delta[t]$.

The authors deem it necessary to note that for nonregular cases similar to that described above for problem (4), when the point $w^0 \in W(t, \vartheta)$ closest to $x_\Delta[t]$ is nonunique, it is necessary to replace, in their paper [11], the procedure described therein for constructing the random motions $x_\Delta[t, t_0, x_0, U^0]$ by the procedure proposed in the present paper. In conclusion we must say that the described scheme of control with a guide $w[t]$ joins the theory of strictly position differential games with the theory proposed by Pontriagin (see [12 − 14]). Both approaches merge into a certain combining stable control scheme based solely on information on the realized states $x[t]$ of the controlled object.

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NECESSARY OPTIMALITY CONDITION FOR THE TIME OF FIRST ABSORPTION

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We examine a linear pursuit problem under conditions of local convexity [1]. We derive the necessary condition for the optimality of the time of first absorption at all points of the space (global optimality). General sufficient conditions for the optimality of the pursuit time have been given in [2, 3].

1. Let a linear pursuit problem in an n-dimensional Euclidean space R be described by:

a) a linear vector differential equation

$$dz/dt = Cz - u + v, \qquad u = u(t) \in P, \qquad v = v(t) \in Q \tag{1.1}$$

where C is a constant square matrix of order n, u and v are vector-valued functions, measurable for $t \geqslant 0$, called the controls of the players (the pursuer and pursued, respectively), $P \subset R$ and $Q \subset R$ are convex compacta;

b) a terminal set M representable in the form $M = M_0 + W_0$, where M_0 is a linear subspace of space R, W_0 is some compact convex set in a space L being the orthogonal complement of M_0 in R .